Radical Expressions

Product Property $\sqrt{a}\cdot\sqrt{b}=\sqrt{ab}$

$\sqrt{20}$ ← radicand

Quotient Property $\frac{\sqrt{a}}{\sqrt{b}}=\sqrt{\frac{a}{b}}$

Simplest Radical Form
- no perfect square factor in the radicand
- no radical in the denominator

1. $\sqrt{50}=\sqrt{2\cdot5\cdot5}=5\sqrt{2}$ (2, 25, 5, 5) / $\sqrt{50}=\sqrt{25}\sqrt{2}=5\sqrt{2}$

2. $\sqrt{7}*\sqrt{35}=\sqrt{7}\cdot\sqrt{7}\cdot\sqrt{5}=7\sqrt{5}$

3. $\sqrt{\frac{11}{144}}=\frac{\sqrt{11}}{\sqrt{144}}=\frac{\sqrt{11}}{12}$

4. $\sqrt{\frac{2}{15}}=\frac{\sqrt{2}}{\sqrt{15}}\cdot\frac{\sqrt{15}}{\sqrt{15}}=\frac{\sqrt{30}}{15}$

5. $4(3+\sqrt{2})=12+4\sqrt{2}$ STOP! $=16\sqrt{2}$ NO!

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Radical Expressions (continued)

6. $(3+\sqrt{2})(4-\sqrt{8})$

FOIL

$12-3\sqrt{8}+4\sqrt{2}-\sqrt{16}$

$12-3\sqrt{8}+4\sqrt{2}-4$

$8-3\sqrt{8}+4\sqrt{2}$

$8-3\sqrt{4}\sqrt{2}+4\sqrt{2}$

$8-3\cdot 2\sqrt{2}+4\sqrt{2} = 8-6\sqrt{2}+4\sqrt{2}$

$= 8-2\sqrt{2}$

7. $\dfrac{4}{5-\sqrt{2}} \cdot \dfrac{5+\sqrt{2}}{5+\sqrt{2}}$ Conjugate

$\dfrac{4(5+\sqrt{2})}{(5-\sqrt{2})(5+\sqrt{2})} = \dfrac{20+4\sqrt{2}}{25+5\sqrt{2}-5\sqrt{2}-2}$

FOIL

$= \dfrac{20+4\sqrt{2}}{23}$ OR $\dfrac{20}{23}+\dfrac{4\sqrt{2}}{23}$

8. $\dfrac{2+2\sqrt{3}}{2} = \dfrac{2}{2}+\dfrac{2\sqrt{3}}{2} = 1+\sqrt{3}$

Complex Numbers

Real

- Counting Numbers $1, 2, 3, 4, \ldots$
- Whole Numbers $0, 1, 2, 3, 4 \ldots$
- Integers $\ldots -3, -2, -1, 0, 1, 2, 3 \ldots$
- Rational Numbers $\frac{p}{q}$ where p & q are integers ($q \neq 0$) (terminating or repeating decimal)
- Irrational Numbers $\pi, \sqrt{2}, \sqrt{3}$

Imaginary

$\sqrt{-1} = i$ or i ~~$-i$~~ $\quad i = \sqrt{-1} \quad i^2 = -1$

$\sqrt{-7} = \sqrt{-1} \cdot \sqrt{7} = i\sqrt{7}$

$\sqrt{-12} = \sqrt{-1} \cdot \sqrt{12} = i\sqrt{12} = i\sqrt{4}\sqrt{3} = i \cdot 2\sqrt{3} = 2i\sqrt{3}$

Solve: $2x^2 + 18 = -72$

$-18 \quad -18$

$2x^2 = -90$

$x^2 = -45$

$\sqrt{x^2} = \sqrt{-45}$

$x = \pm\sqrt{-45}$

$x = \pm i\sqrt{45}$

$= \pm i\sqrt{9}\sqrt{5}$

$= \pm i \cdot 3\sqrt{5} = \pm 3i\sqrt{5}$

Solve: $\frac{-4(n-2)^2}{-4} = \frac{20}{-4}$

$(n-2)^2 = -5$

$\sqrt{(n-2)^2} = \sqrt{-5}$

$\sqrt{-5} = \sqrt{-1} \cdot \sqrt{5}$

$= i\sqrt{5}$

$n - 2 = \pm\sqrt{-5}$

$n - 2 = \pm i\sqrt{5}$

$+2 \quad +2$

$n = 2 \pm i\sqrt{5}$

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Operations with Complex Numbers

Add: $(6 + 3i) - (9 - 2i)$

$6+3i-9+2i$

$-3+5i$

Simplify: $-5i(8 - 9i)$

$-40i+45i^2$

$-40i+45(-1) = -40i-45$

$= -45-40i$

Simplify using FOIL: $(4 + 2i)(3 + 5i)$

$12+20i+6i+10i^2$

$12+26i-10$ $\quad \because i^2 = -1$

$2+26i$

Write in simplest form: Complex Conjugate

$\frac{8}{7+8i} \cdot \frac{7-8i}{7-8i}$

$\frac{8(7-8i)}{(7+8i)(7-8i)} = \frac{56-64i}{49-64i^2}$

$= \frac{56-64i}{49+64}$

OR $\frac{56}{113} - \frac{64}{113}i$

$= \frac{56-64i}{113}$

$\frac{-6-i}{3+10i} \cdot \frac{3-10i}{3-10i} = \frac{(-6-i)(3-10i)}{(3+10i)(3-10i)}$

$= \frac{-18+60i-3i+10i^2}{9-100i^2} = \frac{-18+57i-10}{9+100}$

$= \frac{-28+57i}{109}$